The Interval of Convergence for a Power Series – The Basics

We have already looked at part of this, in lessons 9-2 and 9-3, but now that we know how to test a series for convergence using a variety of tests we have to expand our knowledge of these intervals of convergence.

When we have a power series, it can converge or not just the same as any other series. The **domain** of $f(x)=\sum_{n=0}^{\infty} a_n (x-c)^n$ will be the set of all x for which the series converges.

Every power series will of course converge at its center, when x = c, because when that happens you have the series $f(c)=\sum_{n=0}^{\infty} a_n (c-c)^n = a_n$ (note, we define $0^0 = 1$)

When we find the interval of convergence (and hence the domain of f) one of three different things can happen:

1) The series will converge only for x = c (when the limit diverges),

2) The series converges absolutely for all x (when the limit equals 0), or

3) The series converges absolutely for all x in some open interval of convergence (-R, R) (when the limit is some finite value), and diverges if x lies outside of that interval. This interval, that contains all values of x for which the power series converges is called **the interval of convergence**.

Now comes the tricky part that we have not looked at yet! **It is possible that the power series may also converge at the endpoints of the interval.**

In doing these problems we can find the main interval of convergence (i.e. $|x| < 1$) and then look at the endpoints to see what happens. **Sometimes these series will also converge at the endpoints, so you have to check them.**

This is very important to know for the AP exam! Whether or not you remember to check the endpoints of an interval when you are supposed to is one of those discriminators between 4's and 5's on the exam!!

(1) If the question asks you to find the interval of "absolute" convergence, then you use the same process that we have used before. It is only concerned with the values inside the interval that always produce a convergent series.

(2) But If the question asks you to find the interval of convergence (no mention of only the interval of absolute convergence), then you must also test the endpoints for convergence.

Here is a summary of this process:

How to Test a Power Series for Convergence

1. **Find the interval where the series converges absolutely**. We have to use the Ratio or Root Test unless you have a geometric series, in which case you simply use $|r| < 1$ to get the interval of convergence.

2. If the interval of convergence is finite, also **check the endpoints.** Use a Comparison Test, the Integral Test, or the Alternating Series Theorem.